Chapter 1

Introduction

This thesis is a study of some questions arising from the work of Toyoki Koga (1912-2010) on the foundations of quantum physics. We begin with a few words about Koga’s work.

1.1 Background

Around the turn of the 20th century, investigators like Lorentz, Poincare, Abraham and Mie speculated that the electron’s structure and properties were of electromagnetic origin. This line of thought was abandoned by physicists in the wake of the successes of quantum mechanics from the 1920s onwards.

During the 1950s and 1960s, Toyoki Koga studied the foundations of quantum mechanics with a view to removing ambiguities and contradictions. He was not satisfied with either the Copenhagen interpretation or the work of de Broglie and Bohm. This finally led him to a deterministic theory of the electron including its own internal gravitational field ([9],
Chapter VI of [10]). He then applied this theory to quantum electrodynamics and nuclear physics. (chapters VII-X of [10]). His approach was influenced by, among others, Einstein and the investigators mentioned above.

Before he included gravitation in his theory, Koga gave a treatment of the Schrödinger and Dirac equations for the electron ([13], [14], [12], [7], [8] and Chapters IV and V of [10]). He interpreted his solutions to these equations as localised fields centred around the centre of mass of the electron. He showed that a de Broglie wave for a free electron could be obtained by averaging over an ensemble of solutions to the Schrödinger equation as given by him. This would suggest that the Schrödinger and Dirac theories of Koga are the deterministic theories underlying quantum mechanics which Einstein believed to exist.

Koga studied the Schrödinger equation and the Dirac equation (Chapter V of [10]) but not the Pauli equation. This may be because he found it of no use in developing his general relativistic theory of the electron, including its internal gravitational field (Chapter VI of [10]). The latter was motivated by his solution to the Dirac equation. But the deterministic theory of the Pauli equation is a good illustration of his ideas.

Koga also showed ([8], Chapter V of [10]) that the solution to a system of equations obtained from the Dirac equation could be interpreted in such a way that the Maxwell equations could be derived from them as an approximation under certain conditions.

Koga worked with specific matrix entries and obtained a system of partial differential equations. Without solving the system, he then gave names to certain quantities obtained from the solutions to the PDEs and then interpreted these as the electric field, the magnetic field and so on which appear in Maxwell’s equations. He did not explain how he arrived
at these definitions. It would seem to be difficult or even impossible to get an insight into his derivation.

As a consequence of his approximation procedure Koga obtained the correct value of the magnetic moment of the electron, namely, the Bohr magneton. This showed that his derivation was not an empty mathematical exercise.

Later, Koga ([11], Chapter V) wrote out an explicit solution to the Dirac equation and suggested that the solution represented a spinning field. In [18], a solution to the Dirac equation closely related to Koga’s was given using the Geometric Algebra of David Hestenes [4]. The solution is the sum of three terms: a Klein-Gordon field, a spinning field and another field symmetric about the spin axis.

After Schrödinger discovered the equation named after him, it was found that this equation did not completely describe the electron. It is necessary to ascribe to the electron an intrinsic angular momentum and magnetic moment. This phenomenon is called electron spin since it appears that the electron is spinning.

A non-relativistic theory of the electron, including its magnetic moment, was developed in 1927 by Pauli [21] (see [6] for a modern outline and references to textbooks) who showed how to extend the Schrödinger theory. In the following year, Dirac gave a theory of the electron incorporating special relativity.

It should be noted that by 1970, Dirac had come to believe that a deterministic theory of matter ought to hold ([17], lecture by Dirac).
1.2 Outline of this thesis

In this thesis, we focus on a relatively small but crucial part of Koga’s work: his study of the Schrödinger and Dirac equations, especially their solutions for free electrons.

We first give a brief description of Koga’s solution to the Schrödinger equation (which he called a wavelet in his early papers and an elementary field in his books). Then we discuss and elaborate on his claim that the de Broglie wave associated to a free electron can be obtained by averaging over an ensemble of elementary fields. His treatment of this topic is rather cursory and inadequate, although it is a key part of his work. We give a more detailed explanation.

In the next chapter, we develop the non-relativistic theory of electron spin, as Pauli did, by extending Koga’s solution of the Schrödinger equation. We find that the electron has a definite spin axis at any point of time. An external magnetic field exerts a torque which rotates the spin axis. The Pauli equation holds.

We also discuss the relation of the Hopf map ([5], [20], [15], [19]) to the Pauli spin theory. It has been mentioned by several authors that the Hopf map gives the spin direction of a spin 1/2 particle such as an electron. We show the consistency of this assertion with the Pauli spin theory, which seems to have been taken for granted in the literature so far.

After this, we consider Koga’s solution to the Dirac equation. We show that four one-dimensional solutions to the Klein-Gordon equation each lead to a solution to the Dirac equation containing a term representing a rotating field. Two of these solutions are significant. They represent opposing spins. An electron field with arbitrary spin axis can
be represented as a linear combination of the two. The Hopf map is used in proving this.

Then we continue the study of Koga’s work on the Dirac equation by applying Geometric Algebra to Koga’s approximate derivation of Maxwell’s equations. The notation and methods of Geometric Algebra make the relation between the electromagnetic and Dirac fields easy to see, in fact almost obvious.

We finally summarise the thesis with some concluding remarks.

An appendix dealing with questions posed by an examiner has been added. Errors that he pointed out have been corrected there.