CHAPTER - V

ENERGY GENERATION DURING PRE - SUPERNOVA EVOLUTION
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Introduction

Two of the basic stellar structure equations giving the mass and luminosity of the star are:

\[
\frac{dM_r}{dr} = 4\pi r^2 \rho \quad (5.1)
\]

\[
\frac{dL_r}{dr} = 4\pi r^2 \rho \epsilon \quad (5.2)
\]

Where,

\[
M_r = \text{stellar mass at a distance } r \text{ from the centre}
\]

\[
L_r = \text{luminosity of the star at distance } r
\]

\[
\rho = \text{mass density}
\]

\[
\epsilon = \text{energy production rate}
\]

From equation (1) and (2) we get,

\[
\frac{dL_r}{dM_r} = \epsilon \quad (5.3)
\]

Which describes the photon luminosity per unit mass of the star.

The effective energy production term \( \epsilon \) is given by,

\[
\epsilon = \epsilon_N - \epsilon_u + \epsilon_{wr} \quad (5.4)
\]
Where, \( \varepsilon_N \) = nuclear energy generation rate

\[ \varepsilon_u \left( = \frac{du}{dt} \right) = \text{energy dissipation rate by neutrino emission} \]

\[ \varepsilon_{gr} = \text{gravitational energy release in the star} \]

\( (\varepsilon_N - \varepsilon_u) \) can be taken as the effective nuclear energy generation rate. This term is positive during nuclear burning. Otherwise there will be nothing to balance \( \varepsilon_{gr} \) and contraction will ensue in a star.

Nuclear energy generation takes place due to charged particle interaction at appropriate temperatures inside the star. When the energy completely balances the hydrostatic pressure of the outer layers, the star is said to be in equilibrium. When the nuclear fuel gets exhausted, the gravitational contraction releases energy to counteract the loss of energy from the star.

**Neutrino emission process**

The interaction cross-sections for low energy neutrinos and antineutrinos are so small that neutrinos can escape from the star without any interaction with the material they pass through. Their interaction cross-section with matter is of the order of \( 10^{-44} \text{ cm}^2 \). They have mean free path which depends on the density of the medium. The sun like star has a mean free path of the order of \( 10^{18} \text{ cm} \). For ordinary stars, neutrinos produced,
escape practically with 100 percent probability. However under extreme conditions of condensation, neutrinos will behave differently. The kinetic energy of $\nu$ or $\bar{\nu}$ is lost directly by stars and supernovae without conversion into thermal or electromagnetic energy.

Thus, neutrinos take away energy with them. At low temperature, in $\rho \rho$ chain and CNO cycles neutrinos carry away the available energy with them. However when temperature is greater than $10^9 K$ (temperature range of present work), the rate of energy dissipation by neutrinos become important.

Theoretical and experimental studies of the nature of the weak Fermi interactions, such as beta decay and muon decay indicate that neutrino processes may play an important role in the evolution of stars and the onset of supernova explosions (Fowler and Hoyle, 1964).

Neutrino emission from stars have been treated by Bethe (1939), Gamow and Schonberg (1941), Pontecorvo (1959), Gandel'man and Pinaev (1959), Levine (1960, 1963), Chiu and Morrison (1960), Gell-Mann (1961), Chiu and Stabler (1961), Chiu (1961, 1963), Ritus (1961), Matinyan and Tsilosani (1961), Stothers and Chiu (1962), Sampson (1962), Stothers (1963), Adams, Ruderman and Woo (1963), Rosenberg (1963) and Pinaev (1963) and suggested different neutrino processes.

Three important neutrino processes found that effectively deplete the energy content of highly evolved stars, are:
(i) **Pair annihilation process**  
(Chiu and Morrison, 1960, Levine, 1960)

\[ e^+ + e^- \rightarrow \nu + \bar{\nu} \] (5.5)

The electron - positron pairs are produced at high temperature in stars by the electromagnetic radiation field.

(ii) **Plasma neutrino process** (Adams et al., 1963)

Neutrino-pair emission occurs in the decay of plasmons (\( \gamma_{pl} \)) in a stellar plasma

\[ \gamma_{pl} \rightarrow \nu + \bar{\nu} \] (5.6)

In a dense electron gas, a photon can behave as if it has a rest mass \( \hbar \omega_0 \), \( \omega_0 \) being the plasma frequency. A 'transverse plasmon' as it is then called can decay into a neutrino antineutrino pair. Plasma neutrino process dominates in degenerate matter for which \( \omega_0 > kT/\hbar \). For the same temperature pair annihilation neutrino process dominates over plasma process at densities lower than \( 10^6 \) gm cm\(^{-3} \).
(iii) **Photoneutrino process**

In this process a neutrino pair replaces the scattered photon in photon-electron interaction.

\[ \gamma + e^\pm \rightarrow e^\pm + \nu + \bar{\nu} \]  \hspace{1cm} (5.7)

(Ritus, 1961, Chiu and Stabler, 1961)

Therefore \( e^+ + e^- \rightarrow \nu_e + \bar{\nu}_e \) is by far the most effective of all the neutrino loss mechanisms in massive stars with \( M \geq 10M_\odot \).

Chiu (1961) has calculated the energy loss rate due to pair annihilation process to be

\[ \frac{du}{dt} = \frac{4.8 \times 10^{18}}{\rho} T^3 \exp \left[ -\frac{11.89}{T} \right] \text{erg gm}^{-1} \text{sec}^{-1} \]  \hspace{1cm} (5.8)

\( T \) is in GK(10\(^9\)K)

\( T < 3 \)

\[ \frac{du}{dt} = \frac{4.3 \times 10^5}{\rho} T^9 \text{erg gm}^{-1} \text{sec}^{-1} \]  \hspace{1cm} (5.9)

\( T \) is in GK(10\(^9\)K)

\( T \geq 3 \)

Where \( T \) is the temperature expressed in GK unit and \( \rho \) is density in \( \text{gm cm}^{-3} \).
There are other types of neutrinos which are produced along with tauon and muons. They are more effective at high density and high temperature. In the present case under consideration, they are not supposed to compete with electron neutrinos with regards to energy dissipation. \( \nu_\mu \) and \( \nu_\tau \) carry away 50 - 60% of the \( 2 - 3 \times 10^{53} \) ergs liberated during collapse and explosion. So they have a great role in core collapse of supernovae (Thomson et al., 2000). The core collapse of supernovae are characterized by mass densities of order of \( 10^{10} \sim 10^{14} \) gm/cm\(^3\) and temperature is of the order of 10 GK. Therefore we have not considered \( \nu_\mu \) and \( \nu_\tau \) in the present calculations.

**Nuclear energy generation rate**

The energy generation rate in any thermonuclear reaction is given by

\[
\varepsilon_N = \frac{Q}{\rho} \text{erg gm}^{-1} \text{sec}^{-1}
\]  

(5.10)

Where,

\[ p = \text{reaction rate / cc} \]

\[ \rho = \text{density in gm/cm}^3 \]

\[ Q = \text{disintegration energy expressed in ergs} \]

substituting the expression for \( p \)

\[
\varepsilon_N = \frac{Q}{\rho} \frac{N_1 N_2 \langle \sigma v \rangle}{1 + \delta_{ij}}
\]

\[
N = \frac{\rho \chi}{\Lambda H}
\]

\[
\varepsilon_N = \frac{Q}{\rho} \frac{\rho^2 \chi_1 \chi_2 N_\Lambda^2 \langle \sigma v \rangle}{A_1 A_2} - \frac{Q \rho^2 \chi_1 \chi_2 N_\Lambda N_\Lambda \langle \sigma v \rangle}{A_1 A_2} \text{erg gm}^{-1} \text{sec}^{-1}
\]

\[
\varepsilon_N = \frac{Q \rho \chi_1 \chi_2}{A_1 A_2} \times 10^{23} N_\Lambda \langle \sigma v \rangle \text{erg gm}^{-1} \text{sec}^{-1}
\]

(5.11)
We have considered the same density range given by Fowler and Hoyle (1964) for a polytrope of index 3. Since a $30M_\odot$ star in highly evolved stage can be represented by a polytrop of index 3, the density can be obtained in terms of cubic power of temperature. Their estimation is

$$\rho = 4.8 \times 10^4 T^3 \text{ gm/cm}^3 \hspace{1cm} (T \text{ in GK})$$

**Heavy ion reactions**

After the exhaustion of helium, a core mainly consisting of $^{12}\text{C}$, $^{16}\text{O}$ and $^{28}\text{Ne}$ are produced. No further reaction can take place unless the temperature is considerably raised by contraction. When the temperature increases, the possible reactions are $^{12}\text{C} + ^{12}\text{C}$, $^{12}\text{C} + ^{16}\text{O}$, $^{16}\text{O} + ^{16}\text{O}$, etc.

In present case, a core of either carbon or oxygen is considered. Therefore two reactions are taken into account for energy generation.

(i) $^{12}\text{C} + ^{12}\text{C} \rightarrow ^{24}\text{Mg} + \gamma$

(ii) $^{16}\text{O} + ^{16}\text{O} \rightarrow ^{32}\text{S} + \gamma$

**Calculation**

The energy generation rate of $^{12}\text{C} + ^{12}\text{C}$ and $^{16}\text{O} + ^{16}\text{O}$ reactions are calculated using equation (5.11). The rates are calculated with new reaction rates and present values of $Q$. The results are tabulated in table 5(II). For comparison the energy generation rates of Fowler and Hoyle are also given for these two reactions. Fig. 5.1 shows how these rates vary with temperature.
It is known that as a star evolves its central density and temperature both increases and so also the neutrino energy dissipation rate. The neutrino energy loss is calculated from equations (5.8) and (5.9) and tabulated in table 5(III). Fig. 5.2 gives a plot of energy generation rates along with the energy dissipation rates versus temperature.

Explosive material obtainable for type II supernova is of the order of $2.6M_\odot$. We can have an estimate of the total potential explosive output in a supernova type II when explosive material is either fully $^{12}\text{C}$ or $^{16}\text{O}$. Total energy produced per second during explosive burning of $2.6M_\odot$ material consisting of $^{12}\text{C}$ or $^{16}\text{O}$ are calculated at different temperature and results are tabulated in the table 5.V. Fig. 5.3 shows the trend of energy generation for these temperatures.

In Fig 5.4 another plot was drawn between $(y = ax + b)$ where log $T$ is $x$ and log $E_{cc}$ and log $E_{oo}$ are $y$'s. By least squares fitting the values of the constants are obtained.
**TABLE - 5.1**

\( \mathcal{N}_\mathcal{A} \langle \sigma v \rangle \) values for reactions \(^{12}\text{C} + ^{12}\text{C}\) and \(^{16}\text{O} + ^{16}\text{O}\) and newly calculated \(Q\) values

<table>
<thead>
<tr>
<th>(T) (GK)</th>
<th>(\mathcal{N}<em>\mathcal{A} \langle \sigma v \rangle</em>{cc}) (cm(^3)/s.mole)</th>
<th>(\mathcal{N}<em>\mathcal{A} \langle \sigma v \rangle</em>{oo}) (cm(^3)/s.mole)</th>
<th>(Q_{cc}) (MeV)</th>
<th>(Q_{oo}) (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>.5</td>
<td>(2.94 \times 10^{-20})</td>
<td>(2.67 \times 10^{-38})</td>
<td>12.196</td>
<td>4.47</td>
</tr>
<tr>
<td>.8</td>
<td>(7.89 \times 10^{-14})</td>
<td>(8.87 \times 10^{-28})</td>
<td>12.57</td>
<td>4.94</td>
</tr>
<tr>
<td>1</td>
<td>(3.86 \times 10^{-11})</td>
<td>(2.27 \times 10^{-23})</td>
<td>12.84</td>
<td>5.78</td>
</tr>
<tr>
<td>2</td>
<td>(4.33 \times 10^{-4})</td>
<td>(7.81 \times 10^{-12})</td>
<td>14.80</td>
<td>6.77</td>
</tr>
<tr>
<td>3</td>
<td>(8.64 \times 10^{-1})</td>
<td>(1.73 \times 10^{-6})</td>
<td>15.17</td>
<td>17.16</td>
</tr>
<tr>
<td>5</td>
<td>(1.91 \times 10^{3})</td>
<td>(3.33 \times 10^{-1})</td>
<td>14.94</td>
<td>6.96</td>
</tr>
</tbody>
</table>

**TABLE - 5.2**

Log of energy in different temperature

\(\log \epsilon_{cc}, \log \epsilon_{oo}\) Vs \(T\) (GK)

<table>
<thead>
<tr>
<th>(T) (GK)</th>
<th>(\log \epsilon_{cc _present})</th>
<th>(\log \epsilon_{oo _present})</th>
<th>(\log \epsilon_{cc _FW})</th>
<th>(\log \epsilon_{oo _FW})</th>
</tr>
</thead>
<tbody>
<tr>
<td>.5</td>
<td>.858</td>
<td>-17.3596</td>
<td>0.9164</td>
<td>-17.3017</td>
</tr>
<tr>
<td>.8</td>
<td>7.9122</td>
<td>-6.2117</td>
<td>7.9575</td>
<td>-6.1679</td>
</tr>
<tr>
<td>1</td>
<td>10.9019</td>
<td>-1.4893</td>
<td>10.9377</td>
<td>-1.4691</td>
</tr>
<tr>
<td>2</td>
<td>18.9164</td>
<td>10.9769</td>
<td>18.8907</td>
<td>10.9707</td>
</tr>
<tr>
<td>3</td>
<td>22.7554</td>
<td>16.8607</td>
<td>22.7190</td>
<td>16.8443</td>
</tr>
<tr>
<td>5</td>
<td>26.7588</td>
<td>22.8055</td>
<td>26.729</td>
<td>22.7943</td>
</tr>
</tbody>
</table>
TABLE - 5.III

Variation of Energy dissipation rate with temperature.

<table>
<thead>
<tr>
<th>T (GK)</th>
<th>$\frac{\text{du}_0}{\text{dt}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>.5</td>
<td>$5.0250 \times 10^3$</td>
</tr>
<tr>
<td>.8</td>
<td>$3.7089 \times 10^7$</td>
</tr>
<tr>
<td>1</td>
<td>$7.1936 \times 10^8$</td>
</tr>
<tr>
<td>2</td>
<td>$2.7000 \times 10^{11}$</td>
</tr>
<tr>
<td>3</td>
<td>$6.9547 \times 10^{13}$</td>
</tr>
<tr>
<td>4</td>
<td>$3.9000 \times 10^{14}$</td>
</tr>
<tr>
<td>5</td>
<td>$1.4906 \times 10^{15}$</td>
</tr>
</tbody>
</table>

TABLE - 5.IV

Log T(GK) Vs. Log $\varepsilon_{cc}$, Log $\varepsilon_{oo}$

<table>
<thead>
<tr>
<th>log T (GK)</th>
<th>log $\varepsilon_{cc}$</th>
<th>log $\varepsilon_{oo}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.3010</td>
<td>0.858</td>
<td>-17.3596</td>
</tr>
<tr>
<td>-0.0969</td>
<td>7.9122</td>
<td>-6.2117</td>
</tr>
<tr>
<td>0.00</td>
<td>10.9019</td>
<td>-1.4893</td>
</tr>
<tr>
<td>0.3010</td>
<td>18.9164</td>
<td>10.9769</td>
</tr>
<tr>
<td>0.4771</td>
<td>22.7554</td>
<td>16.8607</td>
</tr>
<tr>
<td>0.6989</td>
<td>26.7588</td>
<td>22.8055</td>
</tr>
</tbody>
</table>
Fig. 5: Plot of $\log \epsilon_c$, $\log \epsilon_0$ versus $T(GK)$.
ENERGY GENERATION RATE & ENERGY DISSIPATION RATE Vs TEMPERATURE

Fig: 5.2
ENERGY PRODUCTION IN $\text{erg sec}^{-1}$ FOR $2.6 M_\odot$ EXPLODING MATERIAL OF $^{12}\text{C}$ OR $^{16}\text{O}$

Fig: 5.3
Fig. 5.4: Plot of \( \log \varepsilon_{cc} \), \( \log \varepsilon_{00} \) versus \( \log T(GK) \). Hollow circles denote \( \log \varepsilon_{cc} \) and filled circles for \( \log \varepsilon_{00} \). Solid line represents least square fitting.
TABLE - 5.V

Energy generation per second during burning of 2.6M☉ exploding material

<table>
<thead>
<tr>
<th>T(GK)</th>
<th>E_{cc} (ergs)</th>
<th>E_{oo} (ergs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>.5</td>
<td>1.2 x 10^{33}</td>
<td>7.45 x 10^{16}</td>
</tr>
<tr>
<td>.8</td>
<td>1.29 x 10^{41}</td>
<td>6.43 x 10^{27}</td>
</tr>
<tr>
<td>1</td>
<td>1.28 x 10^{44}</td>
<td>8.48 x 10^{24}</td>
</tr>
<tr>
<td>2</td>
<td>1.3 x 10^{52}</td>
<td>1.38 x 10^{44}</td>
</tr>
<tr>
<td>3</td>
<td>1.1 x 10^{56}</td>
<td>1.23 x 10^{50}</td>
</tr>
<tr>
<td>5</td>
<td>1.1 x 10^{59}</td>
<td>1.16 x 10^{56}</td>
</tr>
</tbody>
</table>

Results and Discussions

We arrive at two approximate power laws defining temperature dependent energy generation rates for these two reactions.

\[ E_{cc} = 1.1043 \times 10^{10} T_y^{25.8} \]

\[ E_{oo} = 1.1625 \times 10^{-3} T_y^{42} \]

These two laws give us the values of energy generation rates at different temperatures which fairly agree with the calculated values. For \( T=3\text{GK} \) the energy generation values from our formula \( 1.02 \times 10^{56} \text{ ergs sec}^{-1} \) while the calculated value is \( 1.1 \times 10^{56} \text{ ergs sec}^{-1} \)
From the tables and graphs it is seen that at high temperature the \( ^{12}\text{C} + ^{12}\text{C} \) reaction dominates over \( ^{16}\text{O} + ^{16}\text{O} \) reaction.

Because of the change of nuclear mass at higher temperature range, the energy production rate has been found to be much higher.

Thus, it appears that the nuclear mass change is very important at the high temperature situations in astrophysics.

From the tables and graphs it is clear that very high energy can be produced by carbon burning reaction at temperature 2GK but at that temperature oxygen burning cannot produce adequate energy. Perhaps higher temperature (3GK) may be required for oxygen burning to produce this amount of energy.

Thus change of mass with the temperature appears to be very important aspect in explaining many high energy events in astrophysics.
References

- Matinyan and Tsilosani, ZETF. 41, (1681, 1961).