4.1 INTRODUCTION

The study of flows through porous channel has been causing interest amongst the engineers and the mathematicians due to its application in the field of petroleum technology, solid mechanics, ground water hydrology, seepage of water in river beds, purification and filtration processes, civil engineering and bio-mechanics. Berman (1953) presented an exact solution of the Navier-Stokes equations that describes the steady two-dimensional flow of an incompressible viscous fluid along a channel with parallel rigid porous walls, the flow being driven by uniform suction or injection at the walls. Sellars (1953) extended Berman’s work to high suction Reynolds number. Yuan and Finklestein (1956) considered the flow in a porous circular pipe. He obtained solutions for small suction and injection values and asymptotic solution valid at large injection values.

Pulsatile hydromechanics is useful in the study of pressure surges in pipelines, cavitation in hydraulic systems, pumping of slurries and foodstuffs etc. Other areas of technology in which pulsatile flows are important include refrigeration systems, combusting mechanism, de-watering devices and cardiovascular biomechanics. Such flows are characterized by fluctuations in both mass flow rate and pressure about a non-zero mean value. These flows can be generated by a variety of techniques which include reciprocating pumps, hybrid systems with mechanical pulsation devices. Schlichting (1968) has solved the problems related to oscillating flow of fluid under the influence of periodic pressure distribution. Wang (1971) analyzed pulsatile flow in a porous channel. In his investigation, the fluid has been assumed to be driven by a sinusoidal pressure gradient and the velocity of suction at one wall is equal to the velocity of injection at the other.

Rajvanshi (1976) studied viscous flow in a porous channel under pulsating pressure gradient with time dependent suction and injection. The solution has been obtained by separating the steady and unsteady components. The mean velocity in the presence of time dependent porosity and pressure gradient has been studied. He has noted that maxima of the mean velocity profile shifts towards the upper wall with increase in Reynolds number and affected mostly in the vicinity of the walls. Vajravelu et al (2003) have analyzed pulsatile flow of a viscous fluid between two permeable beds. The flow
between and through the permeable beds are governed by the Navier–Stokes equations and Darcy's law, respectively. They discussed velocity field and the volume flux. Their results agree with those of Wang (1971), for the permeability parameter $k \to 0$.

In the recent years, the interest in the effect of Magnetic field on the flow of viscous fluid through a porous channel with ohmic effect has increased. It has been the subject of numerous applications, because most of the liquid metals found in nature are electrically conducting fluids. Singh and Lal (1984) have used Kantorovich method in magnetohydrodynamics flow problems through channels. Formulation of the problem covers a general situation. The solution has been presented for a rectangular channel with non-conducting walls. First two approximations are in agreement with the exact solution. The sequence of successive approximations converges to the exact solution. Malathy and Srinivas (2008) have extended the work of Vajravelu et al (2003) by considering the pulsating flow of a hydromagnetic fluid between two permeable beds. The fluid is injected through the lower permeable bed and is sucked off at the upper permeable bed with the same velocity. A uniform magnetic field in a direction normal to the flow is applied. The effects of magnetic field on the velocity fields are calculated numerically for different values of the parameter. They concluded that the velocity attains its maximum value even at the lower permeable bed in the case of some specific choice of parameters.

The study of heat transfer through porous channel is of much use in the field of Chemical Engineering, Petroleum Engineering, Turbo machinery, Polymer Technology and Aerospace Technology. Transpiration cooling process reduces the heat transfer between the fluid and the walls of the channel. It gives a deeper understanding of the problems of cooling towers and air jet motors. Considerable attention has been given to the study of the problems of heat transfer to pulsatile flow of fluids in channels of various cross-sections due to their increasing applications in the analysis of blood flow and in the flows of other biological fluids. Radhakrishnamacharya and Maiti (1977) have made an investigation of heat transfer to pulsatile flow of a Newtonian viscous fluid in a channel bounded by two infinitely long parallel porous walls. This analysis was carried out to determine theoretically the steady and the fluctuating temperature fields and the rates of heat transfer at the walls. It was shown that the rate of heat transfer at the injection wall which was maintained at temperature $T_0$ increases with the increase of Eckert number.
Ec while at the suction wall which was kept at temperature $T_1 (> T_0)$, heat flows from the fluid to the wall even if $T_1 > T_0$. The combined free and forced convection flow through a porous channel with pulsatile pressure applied across its ends was discussed by Bestman (1982). It was assumed that the ratio of the width of the channel to the length is small. He analyzed that when the channel was horizontal, the axial velocity distribution was exact and independent of the length of the channel, but the pressure and temperature distributions were dependent on the length of the channel. If the channel was taken vertical, then the pressure distribution was obtained exactly. It was independent of the transverse coordinate but varied linearly with the axial coordinate. Heat transfer in unsteady laminar flow through a channel was investigated by Ariel (1990). A similar problem with heat transfer to pulsatile flow of a viscoelastic fluid in a channel bounded by two infinitely long impervious rigid parallel walls was studied by Ghosh and Debnath (1992) with a view to examine its application in the analysis of blood flow. This analysis provides theoretical results for the steady and the unsteady temperature fields and the rates of heat transfer at the walls.

Sharma and Mishra (2002) studied the pulsatile MHD flow and heat transfer through porous channel. Sharma et al (2004) studied unsteady flow and heat transfer of a viscous incompressible fluid between parallel porous disks with heat source/sink. Ghosh and Chakraborty (2005) have studied heat transfer with pulsatile flow of a two-phase fluid-particle system contained in a channel bounded by two infinitely long rigid impermeable parallel walls. The temperature fields were obtained for the fluid and particles. The rates of heat transfer at the walls were also calculated. They observed that the particles have diminishing effects on both the steady and unsteady temperature fields of the fluid and the reversal of heat flux at the hotter wall decreases with the increase of particles irrespective of the influences of other flow parameters.

The effect of suction and injection on the unsteady flow between two parallel disks with variable properties have been studied by Hazem (2005). The viscosity and thermal conductivity of the fluid were assumed to vary with temperature. The disks were kept at constant but different temperatures. The fluid was acted upon by a constant pressure gradient. The coupled set of the nonlinear equations of motion and the energy was solved numerically using the finite difference method. Yadav (2006) studied pulsatile MHD
flow and heat transfer through a porous channel with ohmic effect, when fluid is injected through lower disk and sucked through the upper disk at the same rate. The cross velocity at the disks is taken to be time-dependent. The two disks were maintained at different constant temperatures. The governing equations have been solved by separating steady and unsteady terms. The effect of Hartmann number has been studied on velocity profiles, skin-friction, and Nusselt's number.

In this chapter we have investigated pulsatile MHD laminar flow and heat transfer of an incompressible viscous electrically conducting fluid through non-conducting porous channel in the presence of uniform transverse magnetic field when fluid is injected through lower disk and is sucked through the upper disk at the same rate. The lower disk is kept at constant temperature and a constant heat flux is maintained at the upper disk. Results have been obtained when the upper disk is taken to be adiabatic. The effect of Hartmann number has been discussed on various flow parameters.

### 4.2 Governing Equations

We examine the flow of conducting Newtonian fluid between two long parallel porous non-conducting disks. $x^*$ – axis is assumed to be along the disk. The two disks are assumed to be placed at $y^* = 0$ and $y^* = h$ respectively. The lower disk is kept at constant temperature $T_0$ and upper disk is partially insulated. A uniform transverse magnetic field of intensity $B_0$ is applied. The fluid is injected through the lower disk with velocity $v_0 (1 + e^{i\omega t^*})$ and sucked off at the same rate through the upper disk.

An unsteady pressure gradient

$$-\frac{1}{\rho} \frac{\partial p^*}{\partial x} = A (1 + e^{i\omega t^*}), \quad (4.2.1)$$

is imposed on the fluid, such that $p^*$ is the fluid pressure, $A$ the steady pressure gradient, $\epsilon (< < 1)$ small parameter, $\omega^*$ the frequency of pulsation, $\rho$ the density and $t^*$ is time.
The governing equations are

\[
\frac{\partial v^*}{\partial y^*} = 0 ,
\]

\[
\frac{\partial u^*}{\partial t^*} + v^* \frac{\partial u^*}{\partial y^*} = -\frac{1}{\rho} \frac{\partial p^*}{\partial x^*} + \nu \frac{\partial^2 u^*}{\partial y^*^2} - \frac{\sigma B^2 u^*}{\rho},
\]

\[
\rho \frac{\partial v^*}{\partial t^*} = -\frac{\partial p^*}{\partial y^*} \Rightarrow p^* = f(x^*, y^*, t^*),
\]

\[
\rho C_p \left( \frac{\partial T^*}{\partial t^*} + v^* \frac{\partial T^*}{\partial y^*} \right) = \kappa \frac{\partial^2 T^*}{\partial y^*^2} + \frac{J^2}{\sigma},
\]

where \( u^*, v^* \) are the velocity components along \( x^*, y^* \)-axes respectively, \( \nu \) the kinematics viscosity, \( \sigma \) the electrical conductivity, \( C_p \) the specific heat at constant pressure, \( T^* \) the fluid temperature, \( \kappa \) the thermal conductivity, and \( J \) the conduction current density.

The boundary conditions are

\[
u^* = 0, \quad v^* = v_0(1 + e^{i\omega t^*}), \quad T^* = T_0 \quad \text{at} \quad y^* = 0 ,
\]

\[
u^* = 0, \quad v^* = v_0(1 + e^{i\omega t^*}), \quad \frac{\partial T^*}{\partial y^*} = -\lambda \quad \text{at} \quad y^* = h .
\]
It is common knowledge that the insulation provided in any set-up is not perfect and a small amount of leakage is inherent. In order to take into account that practical situation, the heat transfer at the disk \( y^* = h \) is taken in the form of \( \lambda \). In any efficient set-up this parameter will be very small and has to be determined experimentally. It is expected that this form of the solution will present a more realistic situation.

### 4.3 SOLUTION OF EQUATIONS

For convenience the following non-dimensional quantities are introduced.

\[
x = \frac{x^*}{h}, \quad y = \frac{y^*}{h}, \quad u = \frac{u^*}{(\nu/h)}, \quad v = \frac{v^*}{v_0}, \quad t = \frac{t^*}{(h^2/\nu)}, \quad p = \frac{p^*}{(\rho\nu^2/h^2)}, \quad \theta = \frac{T^*}{T_0},
\]

\[
Pr = \frac{\mu C_p}{\kappa}, \quad R = \frac{v_0 h}{\nu}, \quad M = B_0 h \left( \frac{\sigma}{\rho \nu} \right)^{1/2}, \quad Ec = \frac{\nu^2}{h^2 C_p T_0}, \quad \omega = \frac{\nu \omega^*}{h^2}
\]  

(4.3.1)

where \( Pr \) is the Prandtl number, \( M \) the Hartmann number, \( R \) the cross-flow Reynolds number, \( Ec \) the Eckert number.

Using (4.3.1) into (4.2.2) to (4.2.6), we get

\[
\frac{\partial v}{\partial y} = 0 , \quad (4.3.2)
\]

\[
\frac{\partial u}{\partial t} + R v \frac{\partial u}{\partial y} = - \frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} - M^2 u , \quad (4.3.3)
\]

\[
Pr \left( \frac{\partial \theta}{\partial t} + R v \frac{\partial \theta}{\partial y} \right) = \frac{\partial^2 \theta}{\partial y^2} + M^2 Pr Ec u^2 , \quad (4.3.4)
\]

The boundary conditions in non-dimensional form are reduced to

\[
\begin{align*}
    u &= 0, \quad v = 1 + e^{i\omega t}, \quad \theta = 1 \quad \text{at} \quad y = 0, \\
    u &= 0, \quad v = 1 + e^{i\omega t}, \quad \frac{\partial \theta}{\partial y} = \lambda_i \quad \text{at} \quad y = 1.
\end{align*}
\]

(4.3.5)

where \( \lambda_i = -\frac{\lambda}{T_0} \).

The solution is obtained by separating the steady and unsteady components. The boundary conditions (4.3.5) suggest that the velocity and temperature distributions be
assumed in the form

\[ v = v_0 (1 + \epsilon^{i\omega t}), \quad v_0 > 0, \] (4.3.6)

\[ u(y, t) = A\{u_0(y) + \epsilon u_1(y)\epsilon^{i\omega t} + \epsilon^2 u_2(y)\epsilon^{2i\omega t}\}, \] (4.3.7)

and

\[ \theta(y, t) = \theta_0(y) + \epsilon \theta_1(y)\epsilon^{i\omega t} + \epsilon^2 \theta_2(y)\epsilon^{2i\omega t}. \] (4.3.8)

where \( \omega \) is the non-dimensional frequency.

Using equation (4.3.6), (4.3.7) and (4.3.8) into the equations (4.3.3) - (4.3.4) and equating the coefficients of the like powers of \( \epsilon \), we get

\[ u''_0 - Ru'_0 - M^2 u_0 = -1, \] (4.3.9)

\[ u''_1 - Ru'_1 - (M^2 + i\omega)u_1 = R u'_0 - 1, \] (4.3.10)

\[ u''_2 - Ru'_2 - (M^2 + 2i\omega)u_2 = R u'_1, \] (4.3.11)

\[ \theta''_0 - R Pr \theta'_0 = -Pr M^2 A^2 Ec u_0'^2, \] (4.3.12)

\[ \theta''_1 - R Pr \theta'_1 - i\omega Pr \theta_1 = R Pr \theta'_0 - 2Pr M^2 A^2 Ec u_0 u_1, \] (4.3.13)

\[ \theta''_2 - R Pr \theta'_2 - 2i\omega Pr \theta_2 = R Pr \theta'_1 - Pr M^2 A^2 Ec u_1'^2 - 2Pr M^2 A^2 Ec u_0 u_2, \] (4.3.14)

where prime denote differentiation with respect to \( y \).

The corresponding boundary conditions reduce to

\[ u_0 = 0, \quad u_1 = 0, \quad u_2 = 0, \quad \theta_0 = 1, \quad \theta_1 = 0, \quad \theta_2 = 0 \quad \text{at} \quad y = 0, \]

\[ u_0 = 0, \quad u_1 = 0, \quad u_2 = 0, \quad \theta_0' = \lambda_1, \quad \theta_1' = 0, \quad \theta_2' = 0 \quad \text{at} \quad y = 1, \] (4.3.15)

The solutions of ordinary differential equations from (4.3.9) to (4.3.14) under the boundary conditions (4.3.15) are as follows:

\[ u_0(y) = \gamma_1 e^{n_{1y}} + \gamma_2 e^{n_{2y}} + \frac{1}{M^2}, \] (4.3.16)

\[ u_1(y) = \gamma_3 e^{m_{1y}} + \gamma_4 e^{m_{2y}} + \beta_3 e^{n_{1y}} + \beta_4 e^{n_{2y}} + \frac{1}{M^2 + i\omega}, \] (4.3.17)

\[ u_2(y) = \gamma_5 e^{n_{1y}} + \gamma_6 e^{n_{2y}} + G_1 e^{n_{1y}} + G_2 e^{n_{2y}} + G_3 e^{n_{1y}} + G_4 e^{n_{2y}}, \] (4.3.18)

\[ \theta_0(y) = \gamma_5 + \gamma_6 e^{Pr y} - n_4 e^{2n_{1y}} - n_5 e^{2n_{2y}} - n_6 e^{n_{1y}} - n_7 e^{n_{2y}} - n_8 e^{n_{2y}} + \frac{yEc}{M^2 R}, \] (4.3.19)

\[ \theta_1(y) = \psi_1(y) + i\psi_2(y), \] (4.3.20)

\[ \theta_2(y) = \psi_3(y) + i\psi_4(y), \] (4.3.21)
where

\[ \psi_1(y) = \alpha_1(y) + \alpha_3(y) + \alpha_5(y) + \alpha_7(y) + \alpha_9(y), \]

\[ \psi_2(y) = \alpha_2(y) + \alpha_4(y) + \alpha_6(y) + \alpha_8(y) + \alpha_{10}(y), \]

\[ \psi_3(y) = \alpha_{11}(y) + \alpha_{13}(y) + \alpha_{15}(y) + \alpha_{17}(y) + \alpha_{19}(y) + \alpha_{21}(y) + \alpha_{23}(y) + \alpha_{25}(y) + \alpha_{27}(y) + \alpha_{29}(y) + \alpha_{31}(y) + \alpha_{33}(y) + \alpha_{35}(y) + \alpha_{37}(y) + \alpha_{39}(y) + \alpha_{41}(y) + \alpha_{43}(y), \]

\[ \psi_4(y) = \alpha_{12}(y) + \alpha_{14}(y) + \alpha_{16}(y) + \alpha_{18}(y) + \alpha_{20}(y) + \alpha_{22}(y) + \alpha_{24}(y) + \alpha_{26}(y) + \alpha_{28}(y) + \alpha_{30}(y) + \alpha_{32}(y) + \alpha_{34}(y) + \alpha_{36}(y) + \alpha_{38}(y) + \alpha_{40}(y) + \alpha_{42}(y) + \alpha_{44}(y), \]

\[ \alpha_1(y) = e^{A_{1y}}(D_4 \cos B_4 y - D_2 \sin B_4 y) + e^{A_{1y}}(D_3 \cos B_3 y - D_1 \sin B_3 y) - A_8 e^{2n_1 y} - A_6 e^{2n_2 y} - A_4 e^{n_3 y} - A_4 e^{n_5 y}, \]

\[ \alpha_2(y) = e^{A_{1y}}(D_2 \cos B_3 y + D_1 \sin B_3 y) + e^{A_{1y}}(D_4 \cos B_5 y + D_3 \sin B_5 y) - B_8 e^{2n_1 y} - B_6 e^{n_2 y} - B_4 e^{n_3 y} + n_{13} e^{Pr y}, \]

\[ \alpha_3(y) = e^{A_{1y}}(B_{13} \sin B_{28} y - A_{13} \cos B_{28} y) + e^{A_{1y}}(B_{14} \sin B_{29} y - A_{14} \cos B_{29} y), \]

\[ \alpha_4(y) = -e^{A_{1y}}(A_{13} \sin B_{28} y + B_{13} \cos B_{28} y) - e^{A_{1y}}(A_{14} \sin B_{29} y + B_{14} \cos B_{29} y), \]

\[ \alpha_5(y) = e^{A_{1y}}(B_{18} \sin B_{30} y - A_{18} \cos B_{30} y) + e^{A_{1y}}(B_{19} \sin B_{31} y - A_{19} \cos B_{31} y), \]

\[ \alpha_6(y) = -e^{A_{1y}}(A_{18} \sin B_{30} y + B_{18} \cos B_{30} y) - e^{A_{1y}}(A_{19} \sin B_{31} y + B_{19} \cos B_{31} y), \]

\[ \alpha_7(y) = e^{A_{1y}}(B_{23} \sin B_{2} y - A_{23} \cos B_{2} y) + e^{A_{1y}}(B_{24} \sin B_{3} y - A_{24} \cos B_{3} y), \]

\[ \alpha_8(y) = e^{A_{1y}}(A_{23} \sin B_{2} y + B_{23} \cos B_{2} y) - e^{A_{1y}}(A_{24} \sin B_{3} y + B_{24} \cos B_{3} y), \]

\[ \alpha_9(y) = A_{15} e^{2n_1 y} - A_{16} e^{n_3 y} - A_{17} e^{n_5 y} - A_{20} e^{n_3 y} - A_{21} e^{2n_1 y} - A_{22} e^{2n_2 y} - A_{25} e^{n_3 y} - \frac{2 Ec B_2 y A^2}{\omega} A_{26} e^{n_3 y} + \frac{2 Ec B_2 y A^2}{\omega}, \]

\[ \alpha_{10}(y) = B_{15} e^{2n_1 y} - B_{16} e^{n_3 y} - B_{17} e^{n_5 y} - B_{20} e^{n_3 y} - B_{21} e^{2n_1 y} - B_{22} e^{2n_2 y} - B_{25} e^{n_3 y} - \frac{2 Ec A_2 y A^2}{\omega} B_{26} e^{n_3 y}, \]

\[ \alpha_{11}(y) = e^{A_{1y}}(D_7 \cos B_6 y - D_8 \sin B_6 y) + e^{A_{1y}}(D_5 \cos B_7 y - D_6 \sin B_7 y) - A_{34} e^{2n_1 y} - A_{35} e^{2n_2 y} - A_{36} e^{n_3 y} - A_{37} e^{n_5 y} - n_{36} e^{Pr y} - A_{41} e^{2n_1 y} - A_{42} e^{n_3 y} - A_{43} e^{n_5 y} - A_{46} e^{n_3 y} - A_{47} e^{2n_2 y} - A_{48} e^{n_3 y} - A_{51} e^{n_3 y} - A_{52} e^{n_5 y} - A_{53} e^{2n_2 y} - A_{56} e^{2n_3 y} - A_{63} e^{n_3 y} - \]
\[ A_{65}e^{n_{y}} - A_{67}e^{n_{y}} + B_{57}, \]
\[ \alpha_{12}(y) = e^{A_{y}} \left( D_{8} \cos B_{6}y + D_{7} \sin B_{6}y \right) + e^{A_{y}} \left( D_{6} \cos B_{7}y + D_{5} \sin B_{7}y \right) - B_{34}e^{2n_{y}} - \\
B_{35}e^{2n_{y}} - B_{36}e^{n_{y}} - B_{37}e^{n_{y}} - B_{38}e^{n_{y}} - B_{41}e^{2n_{y}} - B_{42}e^{n_{y}} - B_{43}e^{n_{y}} - B_{46}e^{n_{y}} - \\
B_{47}e^{2n_{y}} - B_{48}e^{n_{y}} - B_{51}e^{n_{y}} - B_{52}e^{n_{y}} - B_{55}e^{2n_{y}} - B_{56}e^{2n_{y}} - B_{57}e^{n_{y}} - \\
B_{67}e^{n_{y}} - A_{57}, \]
\[ \alpha_{13}(y) = e^{A_{y}} \left( A_{32} \cos B_{4}y - B_{32} \sin B_{4}y \right) + e^{A_{y}} \left( A_{33} \cos B_{5}y - B_{33} \sin B_{5}y \right), \]
\[ \alpha_{14}(y) = e^{A_{y}} \left( A_{32} \sin B_{4}y + B_{32} \cos B_{4}y \right) + e^{A_{y}} \left( A_{33} \sin B_{5}y + B_{33} \cos B_{5}y \right), \]
\[ \alpha_{15}(y) = e^{A_{y}} \left( B_{39} \sin B_{28}y - A_{39} \cos B_{28}y \right) + e^{A_{y}} \left( B_{40} \sin B_{29}y - A_{40} \cos B_{29}y \right), \]
\[ \alpha_{16}(y) = -e^{A_{y}} \left( A_{39} \sin B_{28}y + B_{39} \cos B_{28}y \right) - e^{A_{y}} \left( A_{40} \sin B_{29}y + B_{40} \cos B_{29}y \right), \]
\[ \alpha_{17}(y) = e^{A_{y}} \left( B_{44} \sin B_{30}y - A_{44} \cos B_{30}y \right) + e^{A_{y}} \left( B_{45} \sin B_{31}y - A_{45} \cos B_{31}y \right), \]
\[ \alpha_{18}(y) = -e^{A_{y}} \left( A_{44} \sin B_{30}y + B_{44} \cos B_{30}y \right) - e^{A_{y}} \left( A_{45} \sin B_{31}y + B_{45} \cos B_{31}y \right), \]
\[ \alpha_{19}(y) = e^{A_{y}} \left( B_{49} \sin B_{2}y - A_{49} \cos B_{2}y \right) + e^{A_{y}} \left( B_{50} \sin B_{3}y - A_{50} \cos B_{3}y \right), \]
\[ \alpha_{20}(y) = -e^{A_{y}} \left( A_{49} \sin B_{2}y + B_{49} \cos B_{2}y \right) - e^{A_{y}} \left( A_{50} \sin B_{3}y + B_{50} \cos B_{3}y \right), \]
\[ \alpha_{21}(y) = e^{2A_{y}} \left( B_{53} \sin 2B_{2}y - A_{53} \cos 2B_{2}y \right) + e^{2A_{y}} \left( B_{54} \sin 2B_{3}y - A_{54} \cos 2B_{3}y \right), \]
\[ \alpha_{22}(y) = -e^{2A_{y}} \left( A_{53} \sin 2B_{2}y + B_{53} \cos 2B_{2}y \right) - e^{2A_{y}} \left( A_{54} \sin 2B_{3}y + B_{54} \cos 2B_{3}y \right), \]
\[ \alpha_{23}(y) = e^{A_{y}} \left( B_{58} \sin B_{8}y - A_{58} \cos B_{8}y \right) + e^{A_{y}} \left( B_{59} \sin B_{28}y - A_{59} \cos B_{28}y \right), \]
\[ \alpha_{24}(y) = -e^{A_{y}} \left( A_{58} \sin B_{8}y + B_{58} \cos B_{8}y \right) - e^{A_{y}} \left( A_{59} \sin B_{28}y + B_{59} \cos B_{28}y \right), \]
\[ \alpha_{25}(y) = e^{A_{y}} \left( B_{60} \sin B_{30}y - A_{60} \cos B_{30}y \right) + e^{A_{y}} \left( B_{61} \sin B_{2}y - A_{61} \cos B_{2}y \right), \]
\[ \alpha_{26}(y) = -e^{A_{y}} \left( A_{60} \sin B_{30}y + B_{60} \cos B_{30}y \right) - e^{A_{y}} \left( A_{61} \sin B_{2}y - B_{61} \cos B_{2}y \right), \]
\[ \alpha_{27}(y) = e^{A_{y}} \left( B_{62} \sin B_{29}y - A_{62} \cos B_{29}y \right) + e^{A_{y}} \left( B_{63} \sin B_{31}y - A_{63} \cos B_{31}y \right), \]
\[ \alpha_{28}(y) = -e^{A_{y}} \left( A_{62} \sin B_{29}y + B_{62} \cos B_{29}y \right) - e^{A_{y}} \left( A_{63} \sin B_{31}y + B_{63} \cos B_{31}y \right), \]
\[ \alpha_{29}(y) = e^{A_{y}} \left( B_{64} \sin B_{3}y - A_{64} \cos B_{3}y \right), \]
\[ \alpha_{30}(y) = -e^{A_{y}} \left( A_{64} \sin B_{3}y + B_{64} \cos B_{3}y \right), \]
\[ \alpha_{31}(y) = -e^{A_{y}} \left( A_{68} \cos B_{86}y - B_{68} \sin B_{86}y \right) - e^{A_{y}} \left( A_{69} \cos B_{87}y - B_{69} \sin B_{87}y \right), \]

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\[ \alpha_{32}(y) = -e^{\alpha y} (A_{68} \sin B_{66} y + B_{68} \cos B_{66} y) - e^{\alpha y} (A_{69} \sin B_{67} y + B_{69} \cos B_{67} y), \]
\[ \alpha_{33}(y) = -e^{\alpha y} (A_{70} \cos B_{28} y - B_{70} \sin B_{28} y) - e^{\alpha y} (A_{71} \cos B_{29} y - B_{71} \sin B_{29} y), \]
\[ \alpha_{34}(y) = -e^{\alpha y} (A_{70} \sin B_{28} y + B_{70} \cos B_{28} y) - e^{\alpha y} (A_{71} \sin B_{29} y + B_{71} \cos B_{29} y), \]
\[ \alpha_{35}(y) = -e^{\alpha y} (A_{74} \cos B_{88} y - B_{74} \sin B_{88} y) - e^{\alpha y} (A_{75} \cos B_{89} y - B_{75} \sin B_{89} y), \]
\[ \alpha_{36}(y) = -e^{\alpha y} (A_{74} \sin B_{88} y - B_{74} \cos B_{88} y) - e^{\alpha y} (A_{75} \sin B_{89} y + B_{75} \cos B_{89} y), \]
\[ \alpha_{37}(y) = -e^{\alpha y} (A_{76} \cos B_{30} y - B_{76} \sin B_{30} y) - e^{\alpha y} (A_{77} \cos B_{31} y - B_{77} \sin B_{31} y), \]
\[ \alpha_{38}(y) = -e^{\alpha y} (A_{76} \sin B_{30} y + B_{76} \cos B_{30} y) - e^{\alpha y} (A_{77} \sin B_{31} y + B_{77} \cos B_{31} y), \]
\[ \alpha_{39}(y) = -e^{\alpha y} (A_{80} \cos B_{27} y - B_{80} \sin B_{27} y) - e^{\alpha y} (A_{82} \cos B_{22} y - B_{82} \sin B_{22} y), \]
\[ \alpha_{40}(y) = -e^{\alpha y} (A_{80} \sin B_{27} y + B_{80} \cos B_{27} y) - e^{\alpha y} (A_{82} \sin B_{22} y + B_{82} \cos B_{22} y), \]
\[ \alpha_{41}(y) = -e^{\alpha y} (A_{81} \cos B_{37} y - B_{81} \sin B_{37} y) - e^{\alpha y} (A_{83} \cos B_{34} y - B_{83} \sin B_{34} y), \]
\[ \alpha_{42}(y) = -e^{\alpha y} (A_{81} \sin B_{37} y + B_{81} \cos B_{37} y) - e^{\alpha y} (A_{83} \sin B_{34} y + B_{83} \cos B_{34} y), \]
\[ \alpha_{43}(y) = -A_7 e^{2n_y} - A_{75} e^{n_y} - A_{78} e^{n_y} - A_{79} e^{2n_y} - A_{84} e^{n_y} - A_{85} e^{n_y}, \]
\[ \alpha_{44}(y) = -B_{72} e^{2n_y} - B_{75} e^{n_y} - B_{78} e^{n_y} - B_{79} e^{2n_y} - B_{84} e^{n_y} - B_{85} e^{n_y}, \]

where \( A_1 \) to \( A_{89} \), \( B_1 \) to \( B_{89} \), \( A_1^* \) to \( A_5^* \), \( B_1^* \) to \( B_5^* \), \( n_1 \) to \( n_8 \), \( \gamma_1 \) to \( \gamma_6 \), \( \gamma_3^* \), \( \gamma_4^* \), \( G_i \) to \( G_4 \), \( \beta_1 \) to \( \beta_4 \), \( D_1 \) to \( D_5 \), \( g_1 \) to \( g_8 \) are constants recorded in the APPENDIX-IV.

The expressions of \( u(y,t) \) and \( \theta(y,t) \) in the final form are expressed as

\[ u(y,t) = \phi_1(y,t) + i \phi_2(y,t), \]
\[ \theta(y,t) = \phi_3(y,t) + i \phi_4(y,t), \]

where

\[
\phi_1(y,t) = A \left[ \gamma_1 e^{n_y} + \gamma_2 e^{n_y} + \frac{1}{M^2} + \varepsilon \right] + \varepsilon \left\{ \psi_5(y) \cos \alpha t - \psi_6(y) \sin \alpha t \right\} +
\]
\[
\varepsilon^2 \left\{ \psi_5^*(y) \cos 2\alpha t - \psi_6^*(y) \sin 2\alpha t \right\},
\]

\[
\phi_2(y,t) = A \left[ \{ \psi_5(y) \sin \alpha t + \psi_6(y) \cos \alpha t \} + \varepsilon \right] + \varepsilon^2 \left\{ \psi_5^*(y) \sin 2\alpha t + \psi_6^*(y) \cos 2\alpha t \right\},
\]

\[
\phi_3(y,t) = \gamma_1 e^{Pr} - n_4 e^{2n_y} - n_5 e^{n_y} - n_6 e^{-n_y} - n_7 e^{-2n_y} + \frac{y Ec}{M^2 R}
\]
\[
+ \varepsilon \left\{ \psi_1(y) \cos \alpha t - \psi_2(y) \sin \alpha t \right\} + \varepsilon^2 \left\{ \psi_1(y) \cos 2\alpha t - \psi_2(y) \sin 2\alpha t \right\},
\]

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\[ \phi_4(y, t) = e^{\psi_1(y) \sin \omega t + \psi_2(y) \cos \omega t} + e^{\psi_3(y) 2\omega t + \psi_4(y) \cos 2\omega t}, \]
\[ \psi_5(y) = c_{19}e^{A_2y} \cos B_2y - c_{20}e^{A_2y} \sin B_2y + c_{16}e^{A_2y} \cos B_3y - c_{17}e^{A_2y} \sin B_3y + A_4e^{n_2y} \]
\[ + A_5e^{n_2y} + \frac{M^2}{M^4 + \omega^2}, \]
\[ \psi_6(y) = c_{19}e^{A_2y} \sin B_2y + c_{20}e^{A_2y} \cos B_2y + c_{16}e^{A_2y} \sin B_3y + c_{17}e^{A_2y} \cos B_3y + B_4e^{n_2y} \]
\[ + B_5e^{n_2y} - \frac{\omega}{M^4 + \omega^2}, \]
\[ \psi_6^*(y) = c_{19}^*e^{A_2y} \cos B_2^*y - c_{20}^*e^{A_2y} \sin B_2^*y + c_{16}^*e^{A_2y} \cos B_3^*y - c_{17}^*e^{A_2y} \sin B_3^*y + \]
\[ g_1e^{A_2y} \cos B_2^*y - g_2e^{A_2y} \sin B_2^*y + g_3e^{A_2y} \cos B_3^*y - g_4e^{A_2y} \sin B_3^*y + g_5e^{n_2y} + g_7e^{n_2y}, \]
\[ \psi_6^*(y) = c_{20}e^{A_2y} \sin B_2^*y + c_{19}^*e^{A_2y} \sin B_2^*y + c_{16}^*e^{A_2y} \sin B_3^*y + c_{17}^*e^{A_2y} \cos B_3^*y + \]
\[ g_1e^{A_2y} \sin B_2^*y + g_2e^{A_2y} \cos B_2^*y + g_3e^{A_2y} \sin B_3^*y + g_4e^{A_2y} \cos B_3^*y + g_6e^{n_2y} + g_8e^{n_2y}. \]

4.4 RESULTS AND DISCUSSION

The effect of magnetic field and cross flow has been discussed on the various flow parameters. The numerical work has been done for adiabatic case in which we assume \( \lambda_1 = 0 \).

Velocity Distribution

In flow problems the real part of complex quantities has physical significance. Therefore, velocity distribution for \( \omega t = \pi/3 \) is given by
\[ u\left(y, \frac{\pi}{3\omega}\right) = \phi_1\left(y, \frac{\pi}{3\omega}\right), \quad (4.4.1) \]

Fig. 4.2 shows the effect of cross flow Reynolds number \( R \) on velocity distribution. It is seen that fluid velocity decreases with the increase in cross flow Reynolds number \( R \) in the region \( 0 < y < 0.6 \) and it shows different behavior for \( 0.6 < y < 1 \) keeping other parameters constant. Maximum fluid velocity 0.0515 occurs at \( y = 0.52 \) for particular cross flow Reynolds number \( R = 0.5 \).
Fig. 4.3 represents that fluid velocity decreases due to increase in the Hartmann number $M$. It is maximum in the central region. For large value of Hartmann number $M$, fluid velocity shows similar behavior throughout the region. Maximum fluid velocity 0.0628 occurs at $y = 0.59$ for Hartmann number $M = 1.0$. Fig. 4.4 represents the velocity distribution for different values of $\omega t$. These profiles are symmetrical about $y = 0.5$.

**Temperature Distribution**

The temperature distribution for $\omega t = \pi/3$ is given by

$$
\theta \left( y, \frac{\pi}{3\omega} \right) = \phi_y \left( y, \frac{\pi}{3\omega} \right),
$$

(4.4.2)

Fig. 4.5 shows the variation of fluid temperature for different values of cross flow Reynolds number $R$. Fluid temperature decreases as cross flow Reynolds number $R$ increases. Fig. 4.6 shows that fluid temperature decreases as Hartmann number $M$ increases and it increases as Eckert number $Ec$ increases.
Fig. 4.3 Velocity profiles versus $y$ at $\epsilon = 0.25$, $A = 0.5$, $R = 2.0$, $\omega = 4.0$ and $\omega t = \pi/3$.

Fig. 4.4 Velocity profiles versus $y$ at $A = 0.5$, $R = 0.5$, $M = 1.0$ and $Pr = 0.71$. 
Fig. 4.5 Temperature profiles versus \( y \) at \( A = 0.5, M = 2.0, \omega = 4.0 \) and \( \omega t = \pi/3 \).

Fig. 4.6 Temperature profiles versus \( y \) at \( A = 0.5, R = 2.0, \omega = 4.0 \) and \( \omega t = \pi/3 \).
Skin-friction

The skin-friction coefficient at the disks is given by

\[ C_f = \frac{h^2}{\mu U} \left( \frac{\partial y}{\partial y} \right)_{y'=0,h} = \left( \frac{\partial u}{\partial y} \right)_{y=0,1}, \quad (4.4.4) \]

where \( \tau_{x'x'} \) is the shear stress.

Skin-friction coefficients at lower and upper disks respectively, are given by

\[ C_{f_0} = \left( \frac{C_f}{A} \right) = \gamma_1 n_1 + \gamma_2 n_2 + \epsilon \left( c_{19} A_2 - c_{20} B_2 + c_{16} A_3 - c_{17} B_3 + A_1 n_1 + A^*_1 n_2 \right) \cos \omega t - \]

\[ \left( c_{20} A_2 + c_{19} B_2 + c_{16} A_3 + n_1 B_1 + n_2 B_1^* \right) \sin \omega t \] + \epsilon^2 \left( c_{19} A_2^* - c_{20} B_2^* + c_{16} A_3^* - \right.

\left. c_{17} B_3^* + g_1 A_2 - g_2 B_2 + g_3 A_3 - g_4 B_3 + g_5 n_1 + g_7 n_2 \right) \cos 2 \omega t - \left( c_{20} A_2^* + c_{19} B_2^* + c_{16} A_3^* + \right.

\left. c_{17} B_3^* + g_1 B_2 + g_2 A_2 + g_3 B_3 + g_4 A_3 + g_6 n_1 + g_8 n_2 \right) \sin 2 \omega t \]

\[ (4.4.5) \]

and

\[ C_{f_1} = \left( \frac{C_f}{A} \right) = \gamma_1 n_1 e^{n_1} + \gamma_2 n_2 e^{n_2} + \epsilon \left( c_{19} e^{A_2} (A_2 \cos B_2 - B_2 \sin B_2) - c_{20} e^{A_2} (A_2 \sin B_2 + B_2 \cos B_2) + c_{20} e^{A_2} (A_2 \cos B_2 - B_2 \sin B_2) + c_{16} e^{A_2} (A_2 \sin B_2 + B_2 \cos B_2) + \right. \]

\[ A^*_1 n_2 e^{n_2} \} \cos \omega t - \left( c_{16} e^{A_2} (A_3 \sin B_3 + B_3 \cos B_3) + c_{17} e^{A_2} (A_3 \cos B_3 - B_3 \sin B_3) + \right. \]

\left. c_{19} e^{A_2} (A_2 \sin B_2 + B_2 \cos B_2) + c_{20} e^{A_2} (A_2 \cos B_2 - B_2 \sin B_2) + B_1 n_1 e^{n_1} + \right. \]

\left. B^*_1 n_2 e^{n_2} \} \sin \omega t \] + \epsilon^2 \left( c_{19} e^{A_2} (A_2^* \cos B_2^* - B_2^* \sin B_2^*) - c_{20} e^{A_2} (A_2^* \sin B_2^* + B_2^* \cos B_2^*) - \right.

\left. c_{16} e^{A_2} (A_3^* \cos B_3^* - B_3^* \sin B_3^*) - c_{17} e^{A_2} (A_3^* \sin B_3^* + B_3^* \cos B_3^*) + g_1 e^{A_2} (A_2 \cos B_2 - B_2 \sin B_2) - g_2 e^{A_2} (A_2 \sin B_2 + B_2 \cos B_2) + g_3 e^{A_2} (A_3 \cos B_3 - g_4 e^{A_2} (A_3 \sin B_3 + B_3 \cos B_3) + g_5 n_1 e^{n_1} + \right. \]

\left. g_7 n_2 e^{n_2} \} \sin 2 \omega t \right] \]

\[ (4.4.6) \]

Constants occurring here are recorded in the APPENDIX - IV.
Table -1 represents the numerical values of skin-friction coefficient at the lower and upper disks for different values of cross flow Reynolds number $R$ and Hartmann number $M$. The increase in $R$ and $M$ decreases the skin-friction coefficient at the lower disk. The skin-friction coefficient at the upper disk increases numerically due to increase in $R$, while it decreases due to increase in the Hartmann number $M$.

| $R$ | $M$ | $\omega$ | $\omega t$ | $C_{r_0}$ | $|C_{f_i}|$ |
|-----|-----|---------|-----------|---------|--------|
| 0.5 | 2   | 4       | $\pi/3$   | 0.4064  | 0.4821 |
| 1.0 | 2   | 4       | $\pi/3$   | 0.3715  | 0.5282 |
| 2.0 | 2   | 4       | $\pi/3$   | 0.3079  | 0.6644 |
| 2.0 | 1   | 4       | $\pi/3$   | 0.3571  | 0.7906 |
| 2.0 | 3   | 4       | $\pi/3$   | 0.2566  | 0.5403 |
| 2.0 | 5   | 4       | $\pi/3$   | 0.1820  | 0.3836 |
| 2.0 | 10  | 4       | $\pi/3$   | 0.1013  | 0.2850 |

### Nusselt number

The rate of heat transfer in terms of the Nusselt number at the lower disk is given by

$$Nu = \frac{h q^*}{\kappa T_0} = -\left(\frac{\partial \theta}{\partial y}\right)_{y=0},$$

(4.4.7)

where $q^* = -\kappa \left(\frac{\partial T^*}{\partial y^*}\right)_{y^*=0}$.

Hence the Nusselt number at the lower disk is given by

$$(Nu)_0 = -\gamma_6 \Pr R e^{Pr R^y} - 2n_1n_4 e^{2n_1 y} - 2n_2n_5 e^{2n_2 y} - n_4 n_6 e^{n_4 y} - n_5 n_6 e^{n_5 y} - n_1 n_5 e^{n_1 y} +$$

$$\frac{Ec}{M^2 R} + \varepsilon \left\{ \psi'_1(y) \cos \omega t - \psi'_2(y) \sin \omega t \right\} + \varepsilon^2 \left\{ \psi'_3(y) \cos 2 \omega t - \psi'_4(y) \sin 2 \omega t \right\}_{t=0},$$

(4.4.8)
Table -2 represents the numerical values of Nusselt number at the lower disk for different values of cross flow Reynolds number $R$, Hartmann number $M$. The increase in $R$ and $M$ decreases the value of Nusselt number, while it increases as Eckert number $Ec$ increases.

| $R$ | $M$ | $\omega$ | $\omega t$ | Ec | $|Nu_0|$ |
|-----|-----|----------|-----------|----|--------|
| 0.2 | 2   | 4        | $\pi/3$   | 0.01 | 0.0013 |
| 0.5 | 2   | 4        | $\pi/3$   | 0.01 | 0.0011 |
| 2.0 | 2   | 4        | $\pi/3$   | 0.01 | 0.0006 |
| 0.5 | 0.5 | 4        | $\pi/3$   | 0.01 | 0.0175 |
| 0.5 | 1   | 4        | $\pi/3$   | 0.01 | 0.0044 |
| 0.5 | 1.5 | 4        | $\pi/3$   | 0.01 | 0.0020 |
| 0.5 | 0.5 | 4        | $\pi/3$   | 0.02 | 0.0350 |

4.5 CONCLUSION

1. The fluid velocity decreases due to increase in cross flow Reynolds number $R$ and Hartmann number $M$.
2. The fluid temperature decreases due to increase in cross flow Reynolds number $R$, Hartmann number $M$, while it increases as Eckert number $Ec$ increases.
3. The skin-friction coefficient at the lower disk decreases due to increase in cross flow Reynolds number $R$ and Hartmann number $M$, while it increases numerically at the upper disk due to increase in cross flow Reynolds number $R$ and Hartmann number $M$.
4. The Nusselt number at the lower disk decreases numerically as cross flow Reynolds number $R$ and Hartmann number $M$ increases, while it increases as Eckert number $Ec$ increases.